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Brief Descriptions of Quantitative Research Methods

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In this chapter, an assortment of quantitative research methods is illustrated. They range from Achievement Testing to Unidimensional Scaling. They are presented separately in alphabetical order for the sake of lucidity.

Achievement Testing

Achievement tests are used to evaluate both written and practical knowledge. Thus, Achievement Testing Methodology is used to assess strengths and weaknesses of an individual's aptitude and capabilities (Wechsler 2005). The methodology is used concurrently with intelligence tests to compare and identify shortcomings and strengths of individuals in order to initiate a process of addressing their weaknesses and reinforcing the identified strengths (Frederick and Markwardt 1989). Intelligence tests assess the cognitive and problem-solving skills of an individual; thus, they illustrate their intellectual potential (Psychological Cooperation, 1992). An example of the application of Achievement Testing Methodology is a class test for high school or college students. Giving a driving road test to get a driver's license is also another example of applying the Achievement Testing Methodology.

Achievement and intelligent tests are usually the initial discriminatory tests applied in evaluating development. They are used in development sometimes in the form of psychometric tests (Nizoloman 2013). For example, a psychologist can use verbal comprehension in an IQ test associated with reading skills on an achievement test. Identified challenges in assessed areas may imply knowledge challenges and, thus, more testing may be suggested (Domino and Domino 2006). Examples of commercialized achievement tests include Wechsler Individual Achievement Test (WIAT-III) and the Peabody Individual Achievement Test. The WIAT- III is used for individuals between 4-85 years of age to assess reading, writing, speaking and mathematical aptitude and capabilities (Wechsler 2005). The Peabody Individual Achievement Test is used for individuals between the ages of 5-22 years of age in assessing learning disabilities and the appraisal of programs.

The Achievement Testing Methodology was employed in the study of Masino and Nino-Zarazua (2016) to assess and select policy interventions in education using a sample from developing countries. A study by Nizoloman (2013) also utilized the methodology to establish mathematical ability and achievement among secondary school students using a Nigerian sample.

Agent-Based Models (ABMs)

Agent-based modeling is a computational method that enables a researcher to develop, analyze, and experiment with models composed of agents that interact within an environment (Gilbert 2007). A model is intended to represent or simulate some real, existing phenomenon, and this is called the target of the model. Agents are either separate computer programs or, more commonly, distinct parts of a program that are used to represent social actors—individual people, organizations such as firms, or bodies such as nation-states. They are programmed to react to the computational environment in which they are located, where this environment is a model of the real environment in which the social actors operate. The environment is the virtual world in which the agents act. It may be an entirely neutral medium with little or no effect on the agents or, in other models, the environment may be as carefully crafted as the agents themselves.

Bonabeau (2002) describes agent-based modelling as a powerful simulation technique that grew incrementally and now includes several applications, including applications to real-world business problems. Gilbert (2007) argues that by using Agent-Based Models (henceforth, ABMs), one creates a simplified representation of "social reality" that serves to express as clearly as possible the way in which one believes that reality operates. For example, if one has a dependent variable and one or more independent variables, a regression equation serves as a model of the relationship between the variables. A network of nodes and edges can model a set of friendships.

ABMs are most often used as computerized abstractions/ideas of something from the real world or something that exists in conceptual form (Gilbert 2007). The agents in an ABM are almost always used to process data. ABMs thus act as a surrogate representation of some phenomenon or system for the purposes of experimentation or scenario-building. Bonabeau (2002) states that ABMs offer more benefits compared to other modelling techniques because of (a) their ability to capture emergent phenomena, (b) they provide a natural description of a system, and (c) their flexibility. As an example, from an investor's point of view, ABMs represent a new direction that may or may not result in more practical investment models. From an academic's point of view, however, ABMs are fascinating new tools that make it possible to investigate problems previously out of reach.

Gilbert (2007) distinguishes between individual-based and multi-agent-based models. According to him, individual-based models are usually designed to represent the behavior of a single agent such as an institution, a mind, or a predatory animal, while multi-agent models are generally built with many individual agents, each of which may play a different role or assume a set of distinct tasks in the model. Such models generally focus on the interactions among individuals or units. Multi-agent systems adopt a synoptic view, often from the bottom-up, to consider individual agency in the context of a larger or collaborative phenomenon. Often, the system that is considered is treated as a complex adaptive system and the interplay between agents and agent attributes is used to explore issues of emergence, feedback, self-organization, phase shift, and so on.

Gilbert (2007) also suggests that ABMs are very popular for their use in geographical research as a methodology for simulating risk and vulnerability. Emergency and catastrophic events, by their very nature, are all but impossible to explore and experiment in the real world. Models are therefore invaluable tools for evaluating plans and scenarios for vulnerability and resilience. Such models need to be realistic to be useful; however, ABMs can play an important role in modelling the human agency in panic, evacuation, and response.

Bootstrapping

The Bootstrapping Method was first proposed in 1979 by Bradley Efron. It is a statistical method used to obtain a nonparametric estimate of standard error, biases, and confidence interval (Efron 1979). The method consists of resampling some given observations to obtain a good estimate of statistical properties of the original population. According to Ujeh et al. (2016), bootstrapping is a resampling technique to estimate sample statistics with precision to perform significance tests and to validate statistical models in situations where theoretical inference is difficult to get the result directly, especially in finding variance estimates.

Since its invention, bootstrapping has become a very powerful tool used to improve statistical estimation, especially in situations where the data are not normally distributed (Chaitip, Chaibonsi and Inluang 2014). Bootstrapping is also applicable for sample mean, sample quantile, t-statistics, and linear combination of order statistics (Chao and Lo 1985). It provides more accurate point estimates for prediction error and the variability for estimated parameters that can easily be assessed. It also provides a better way to compute empirically relevant critical values for hypothesis tests in econometrics.

Due to the problem of misspecification, a new group of bootstrapping techniques have been advanced to reduce this risk. Examples include the block, sieve, and local methods of bootstrapping. and these techniques are mostly nonparametric (Cerqueti, Falbo and Pelizzari 2013). Univariate distribution samples have also been generated using the Bootstrapping Method. This method treats the population distribution as an unbiased estimate. Nonetheless, given a multivariate distribution setting, the researcher must be careful with bootstrapping especially where dependence among variables is reproduced (Ruscio et al. 2007).

Dohan, Schmidt and Henderson (2005) used the Bootstrapping Method to study welfare workers' views on substance abuse and welfare reform. The authors found out that some workers endorse the notion of bootstrapping defined by the *American Heritage Dictionary* as acting "by the use of one's own initiative and work without reliance on outside help." (Date?) These workers adhered to the belief that individual orientations, attitudes, and behaviors made the crucial difference in whether welfare recipients would succeed in fulfilling their welfare contracts. In another study, to test and measure the efficiency and changes in productivity in the United Kingdom airline industry, Assaf (2011) used the Bootstrap Malmquist Index Method to overcome the statistical limitations of the data.

In yet another illustration, the Bootstrapping Method was employed to indicate the statistical correctness of the panel data Chaitip and his colleagues (2014) used to improve the estimation accuracy of their study. The study examined the statistical properties of technical efficiency in a panel data setting using data for sugarcane farming households in Thailand. Bootstrapping has also been used to explain the concept of learning. Beck (2017) employed the Bootstrapping Method to explain the concept of learning. He distinguishes two versions of bootstrapping called (1) modest bootstrapping and (2) radical bootstrapping.

In another study, Basiri et al. (2017) utilized the Bootstrapping Method to perform statistical inference on the FastICA (a popular algorithm for independent component analysis) estimates of mixing matrix. They used this method because bootstrapping provides an important tool to test hypotheses, providing an important assessment of the quality of estimators in terms of confidence intervals, standard error, and variance.

Calculus for Social Science Research

According to Stewart (2011), Calculus is divided into two branches: (1) Differential Calculus and (2) Integral Calculus. Differential Calculus uses the concept of function derivative to analyze behavior and the rate of change. Using Differential Calculus, we can compute, analyze, and predict the graph of functions. Integral Calculus is the reverse process of differentiation, concerned with the concept of the anti-derivative. Literally, integration means accumulation.

Calculus is highly utilized in the social sciences where it is used to develop mathematical models to arrive at an optimal solution. For example, in Economics, Calculus is used to compute marginal cost and marginal revenue, enabling economists to predict maximum profit in a specific setting.

As mentioned earlier, Calculus is divided into Differential and Integral Calculus. Differential Calculus is the rate of change of a quantity (Stewart 2011; Hughe-Hallet et al. 2009); thus, it is how fast a quantity is changing. Differentiation is usually perceived as the slope of a tangent line to a function (Boyer 1991) and calculated by finding the limit as h approaches zero of the quotient of the difference of the value of the function at a point $x = a + h$ and the value of the function at the point $x = a$ with the value of h. Mathematically, this is expressed as follows:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Integral Calculus tries to estimate the area under the curve by using approximating rectangles. The number of rectangles used in the computation is n and, in the definite integral, the number of rectangles is allowed to approach infinity. As n approaches infinity, the width of the rectangles is forced to shrink towards zero and results in the approximating area becoming exact (Stewart 2011).

Mathematically, the definite integral is the limit as n approaches infinity of the sum from 1 to n of the product of the function with the change in x:

$$\int_a^b f(x)dx = \lim_{h\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

In social science research, several researchers have made use of Calculus. For example, Biagini and Ø ksendal (2005) made use of stochasticity to model insider trading. The study modelled the market which is influenced by many investors with insider information. Tejado et al. (2015) also modelled economic growth using fractional Calculus and the Spanish data.

Chaos and Catastrophe Theories

The Chaos and Catastrophe Theories Method originated from the fields of Mathematics and Physics. Chaos Theory is a rigorous mathematical model also known as the butterfly effect (Kiel and Welliott 1997). The model aims to explain how minor modifications in initial conditions within complex dynamic structures can extensively affect outcomes (Lorenz 1996.). It is used to explain the elements affecting traffic flow in traffic jams, weather prediction models, and epileptic fits among other factors. Chaos Theory has been used in Computer Science to refine robotics through predictive models to eliminate the trial and error process in interacting with the environment (Ulrich and Walker 2005). It is also used in Psychology to support notions that an individual replicates group dynamics on a different scale, and the chaotic behavior of the group is reflected in each member (Forno and Merlone 2013).

Catastrophe Theory refers to the model of an abrupt discontinuous transition of phenomenon such as the fall of a government, a riot by a mob, and the freezing of water or other liquids (Zeeman 1977; Igorevich 1992). It is used to show a sequence of continuous changes: for example, how the frustration of citizens in a country can initiate quick and broad scale changes (catastrophic adjustments) such as the removal of a leader or the fall of an empire (Thom 1989). The removal of President Park Geun-hye of South Korea by the citizens as a result of her abuse of power and corruption (Aljazeera 2017) is an example. Another example can be the collapse of a bridge due to harsh weather conditions. This can be related to the Nkankezi Bridge in Zimbabwe in the Masvingo-Bulawayo highway which collapsed because of Cyclone Dineo in February 2017 (*Chronicle* 2017). Catastrophe Theory is therefore a way for a continuous function to model an abrupt change that would normally be called a discontinuity (Zeeman 1977; Bangura 2012).

Chaos Theory was utilized in the study of Adewumi et al. (2016) to predict motorized traffic flows on urban networks in South Africa. The results of the study addressed the challenges of traffic jams through the identification of techniques which are effective in controlling traffic flow. The model was also utilized in a tourism study in which ecotourism was conceptualized as a change agent in African countries (Ariwa and Syvertsen 2010). The results of the study were used to promote tourism in the continent. Catastrophe Theory was employed in a study by Birau (2013) for market forecasting in Africa. The theory yielded a new level of understanding concerning the idea of the stock market. Kaki (2012) in his study also utilized Catastrophe Theory to highlight the catastrophic behavior of nerve cells.

Quantitative Content Analysis

The Quantitative Content Analysis Method generally refers to the 'systematic counting, assessing, and interpreting of the form and substance of communication' (Manheim and Rich 1981:155) The method utilizes a variety of tools to study media content. The broad nature of the field has led to various definitions over the years. According to Berelson, content analysis is 'a research technique for the objective, systematic, and quantitative description of the manifest content of communication' (1952:18). Holsti says that content analysis is 'any technique for making inferences by systematically and objectively identifying specified characteristics of messages' (1968:14) Kerlinger defines content analysis as 'a method of studying and analyzing communication in a systematic, objective, and quantitative manner for the purpose of measuring variables' (1986:523). More modern definitions have specifically included references to social media, sentiment analysis, and big data approaches. Overall, the Quantitative Content Analysis Method in this way transforms observations of found categories into quantitative statistical data.

Content analysis grew out of work conducted by theorist Alfred Lindesmith, who devised a means of refuting a hypothesis known as "The Constant Comparative Method of Qualitative Analysis" in 1931. Quantitative analysis built upon these qualitative research tools and applied more rigorous statistical and scientific techniques. Krippendorff (2004) developed the following six questions based on Lind Smiths' work that must be considered in any content analysis:

1. Which data are analyzed?
2. How are they defined?
3. What is the population from which they are drawn?

4. What is the context relative to which data are analyzed?
5. What are the boundaries of the analysis?
6. What is the target of the inferences?

When analyzing data using the Quantitative Content Analysis method, the assumption is that words and phrases mentioned most often are those reflecting important concerns in every communication. Therefore, the method starts with word frequencies, space measurements (column centimeters/inches in the case of newspapers), time counts (for radio and television time) and keyword frequencies. Nonetheless, the method extends far beyond plain word counts. For example, with "Keyword in Context" routines, words can be analyzed in their specific context to be disambiguated.

Quantitative Content Analysis requires formal properties such as word frequencies, space measurements, time counts, hashtags, number of tagged people in an image, number of friends, or liked pages. The objects of analysis may vary from traditional textual content (messages, bibliometric, citation analysis/indexing, webpages, trending topics on twitter), to any media object with specified formal properties or metadata (video, photographs, phone conversations). At least three important distinctions from qualitative content analysis arise because of this. First, as opposed to qualitative analysis, quantitative (or computer-based and automated) analysis is better suited for closed inquiries, and typically results in emergent categories rather than manually assigned categories (which also make this type of analysis useful to derive probable predictions about the future). Second, because of focusing only on formal properties, quantitative content analysis typically applies to manifest contents (literal content) rather than its latent meaning (implied content). Third, McKeone (1995) distinguishes between prescriptive analysis (which has a closely defined set of specific parameters) and open analysis (which can be applied to many types of texts and content, and where dominant messages are identified in the analysis). Moreover, because the researcher often requires instruments to measure and count (e.g., a computer), the reliability (every research will get the same results) and validity (it measures what it is supposed to measure) of the apparatus and techniques (e.g. its software) should always be reflected upon as part of the research (Zeh 2005).

Expert Systems

An expert system is a computer program that simulates the thought process of a human expert to solve complex decision problems in a specific domain. The growth of expert systems is expected to continue. With the continuing growth, many new and exciting applications will emerge. An expert system operates as an interactive system that responds to questions, asks for clarification, makes recommendations, and generally aids the decision-making process. Expert systems provide expert advice and guidance in a wide variety of activities, from computer diagnosis to delicate medical surgery (Jackson 1999; Bullinaria 2005). Various definitions of expert systems have been offered by several authors.

A general definition that is representative of the intended functions of an Expert Systems Method is that it is an interactive computer-based decision tool that uses both facts and heuristics to solve difficult decision problems based on knowledge acquired from an expert. An expert system may equally be viewed as a computer simulation of a human expert. Jackson (1999) provides us with the following definition: "An *expert system* is a computer program that represents and reasons with knowledge of some specialist subject with a view to solving problems or giving advice. To solve expert-level problems, expert systems will need efficient access to a substantial domain *knowledge base* and a *reasoning mechanism* to apply the knowledge to the problems they are given" (Jackson 1999: 5). Usually they will also need to be able to explain to the users who rely on them how they have reached their decisions. They will generally build upon the ideas of knowledge representation, production rules, search, and so on (Jackson 1999; Bullinaria 2005).

Past applications of expert systems range from MYCIN (an expert system designed to provide expert level solutions to complex problems), used in the medical field to diagnose infectious blood diseases, to XCON (developed by Digital Equipment Corporation), used to configure computer systems. These expert systems have proven to be quite successful. Most applications of expert systems will fall into one of the following categories: interpreting and identifying, predicting, diagnosing, designing, planning, monitoring, debugging and testing, instructing and training, or controlling (Jackson 1999; Bullinaria 2005).

Expert Systems are designed by knowledgeable engineers to simulate the type of reasoning, decision-making and cognitive processes that an expert in a given field or occupation would demonstrate. There are two primary parts to expert systems: (1) a knowledge base and (2) a reasoning engine. The knowledge base contains both factual and judgmental knowledge. The reasoning engine uses inference to solve problems, often using "if-then" decision chains (Jackson 1999; Bullinaria 2005).

One example of an Expert System is an artificial intelligence system that emulates an auto mechanic's knowledge in diagnosing automobile problems**.** This hypothetical expert system would likely be the result of engineering using an actual mechanic's knowledge base. Some typical existing Expert System tasks include the interpretation of data such as sonar data or geophysical measurements, diagnosis of malfunctions such as equipment faults or human diseases, structural analysis or configuration of complex objects such as chemical compounds or computer systems, planning sequences of actions such as might be performed by robots, and predicting the future such as weather, share prices, exchange rates (Jackson 1999; Bullinaria 2005).

Graph Algebra

Graphs are combinatorial objects that sit at the core of mathematical intuition. They appear in numerous situations throughout mathematics and have often constituted a source of inspiration for researchers (Pino, et al. 2006). Humpert and Martin (2011) describe Graph Algebra as a commutative, co-commutative, graded, connected Hopf Algebra (construct related to the symmetries of group actions), whose basic elements correspond to finite graphs, and whose Hopf product (solution) and co-product admit simple combinatorial descriptions.

Graph Algebra is integrated with discrete time application, which implies the use of different equations, which in turn are often appropriate for the social sciences since a great deal of social scientific data is collected in discrete intervals. Examples of these are census data, economic data, election data, and polling data (which often correspond with an electoral calendar). Models can also be built by using Graph Algebra that have both continuous and discrete parts. These are called metered differential equations. Graph Algebra is a language that can be used in an explanatory manner to develop new nonlinear model specifications that may be associated with chaos and other complex behaviors. For example, perhaps the most famous of all continuous time models used to study chaos involves "strange attractors" discovered by Lorenz (1963).

When developing a Graph Algebra model, a theorist is confronted with the challenging problem of having to figure out, on the level of algebra alone, how change in one variable will be affected by both the levels and the change in other variables. Many social theorists may find it helpful when working with nonlinear processes to exploit a Graph Algebra schematic picture of these processes, thereby extending the algebraic flexibility of their efforts (Brown 2008).

Internet Data Collection

Due to the advancement of information and communication technologies (ICTs), new technological tools have emerged to support data collection for the production of quality research (Benfield and Szlemko 2006). The Internet Data Collection Method has become a tool utilized by many researchers to access quality research materials. The high demand for Internet-based data collection technique is occasioned by the ease with which information is obtained and utilized or transferred by the use of electronic resources (Rosnow and Rosenthal 2005).These electronic resources include software and databases used for data collection and analysis.

The first case of electronic survey called the current mining and manufacturing survey was applied in South Korea in 1977. The survey was conducted within the framework of Internet-based data collection. In the process of data collection, software for an electronic survey was installed in the computer(s) of a respondent within an establishment (NSO Report 2006). Upon further education on how to use the program, respondents were able to fill out the electronic questionnaire using the software at their convenience after which the questionnaires were then transferred to the file of the National Statistics Office (henceforth, NSO). NSO examined the submitted files and summed them up with results in a paper survey to produce its total statistics.

The Internet-based electronic survey is advantageous in that it saves time and is less costly since Internet services reduce the possibility of over-employment and, hence, maximizing profit for the institution. Even though the Internet Data Collection Method brought positive innovations towards data collection and subsequent analysis, it recorded alongside disadvantages associated with database optimization. In cases where participants witnessed a faulty installation or encountered error in the use of databases like Statistical Package for Social Sciences (SPSS), NetLogo, NVivo etc., they had no option but to uninstall and reinstall the database to ensure proper functioning (Koo and Skinner 2005). This process is not only time consuming, but it also leads to inconsistence for inexperienced researchers. Also, outdated computers are unable to support new software programs, thereby propagating innovative research. Additionally, it seems impossible to get assistance in situations where respondents are inexperienced. In attempting to solve the problems associated with Internet-based data collection, manual installations of software used for data collection and analysis were replaced with automatic databases available online. This innovative approach reduced the difficulties encountered with faulty installations and the use of outdated software. Also, respondents could now access institutional servers using specific identification passwords to access data and fill out electronic questionnaires within institutional data bases (Koo and Skinner 2005).

Linear Programming

The development of the Linear Programming Method has been ranked among the most important scientific advances of the mid-20th Century. Its impact since 1950 has been extraordinary. Today, it is a standard tool that has saved many millions of dollars for most companies or businesses of even moderate size in the various industrialized countries of the world, and its use in other sectors of society has been spreading rapidly. A major proportion of all scientific computations on computers is devoted to the use of linear programming. Dozens of textbooks have been written about the method, and published articles describing important applications now number in the hundreds (Hillier and Lieberman 2001). What is the nature of this remarkable tool, and what kinds of problems does it address?

Briefly, the most common type of application for linear programming involves the general problem of allocating *limited resources* among *competing activities* in a best possible (i.e. *optimal*) way. More precisely, this problem involves selecting the level of certain activities that compete for scarce resources that are necessary to perform those activities. The choice of activity levels then dictates how much of each resource will be consumed by each activity. The variety of situations to which this description applies is diverse, indeed, ranging from the allocation of production facilities to the allocation of national resources to domestic needs, from portfolio selection to the selection of shipping patterns, from agricultural planning to the design of radiation therapy, and so on. However, the one common ingredient in each of these situations is the necessity for allocating resources to activities by choosing the levels of those activities (Hillier and Lieberman 2001).

The Linear Programming Method uses a mathematical model to describe the problem of concern. The adjective *linear* means that all the mathematical functions in this model are required to be *linear functions.* The word *programming* does not refer here to computer programming; rather, it is essentially a synonym for *planning.* Thus, linear programming involves the *planning of activities* to obtain an optimal result: i.e. a result that reaches the specified goal best (according to the mathematical model) among all feasible alternatives. Although allocating resources to activities is the most common type of application, linear programming has many other important applications as well. In fact, *any* problem whose mathematical model fits the very general format for the linear programming model is a linear programming problem. Furthermore, a remarkably efficient solution procedure, called the *simplex method*, is available for solving linear programming problems of even enormous size. These are some of the reasons for the tremendous impact of linear programming in recent decades (Hillier and Lieberman 2001).

An example of how the Linear Programming Method has been utilized is how the Wyndor Glass Company produces high-quality glass products, including windows and glass doors. It has three plants: aluminum frames and hardware are made in Plant 1, wood frames are made in Plant 2, and Plant 3 produces the glass and assembles the products. Because of declining earnings, top management decided to revamp the company's product line. Unprofitable products were discontinued, thereby releasing production capacity to launch two new products having large sales potential: Product 1, an 8-foot glass door with aluminum framing and Product 2, a 4x6 foot, double-hung wood-framed window. Product 1 requires some of the production capacity in Plants 1 and 3, but none in Plant 2. Product 2 needs only Plants 2 and 3. The marketing division concluded that the company could sell as much of either product as could be produced by these plants. Nonetheless, since both products would be competing for the same production capacity in Plant 3, it was not clear which *mix* of the two products would be *most profitable.* Therefore, an operations research (henceforth, OR) team was formed to study this question. The OR team began by having discussions with upper management to identify management's objectives for the study. These discussions led to the following objective: Determine what the *production rates* should be for the two products in order to *maximize their total profit,* subject to the restrictions imposed by the limited production capacities available in the three plants. (Each product will be produced in batches of 20, so the *production rate* is defined as the number of batches produced per week.) *Any* combination of production rates that satisfies these restrictions is permitted, including producing none of one product and as much as possible of the other (Hillier and Lieberman 2001).

The OR team also identified the data that needed to be gathered: (a) number of hours of production time available per week in each plant for these new products (most of the time in these plants is already committed to current products, so the available capacity for the new products is quite limited); (2) number of hours of production time used in each plant for each batch produced of each new product; (3) profit per batch produced of each new product. (*Profit per batch produced* was chosen as an appropriate measure after the team concluded that the incremental profit from each additional batch produced would be roughly *constant* regardless of the total number of batches produced (Hillier and Lieberman 2001). Since no substantial costs will be incurred to initiate the production and marketing of these new products, the total profit from each one is approximately this *profit per batch produced* times *the number of batches produced*(Hillier and Lieberman 2001).

Obtaining reasonable estimates of these quantities required enlisting the help of key personnel in various units of the company. Staff in the manufacturing division provided the data in the first category. Developing estimates for the second category of data required some analysis by the manufacturing engineers involved in designing the production processes for the new products. By analyzing cost data from these same engineers and the marketing division, along with a pricing decision from the marketing division, the accounting department developed estimates for the third category. The OR team immediately recognized that this was a linear programming problem of the classic *product mix*type, and the team then undertook the formulation of the corresponding mathematical model (Hillier and Lieberman 2001).

Matrix Algebra

Algebra is a part of mathematics that deals with operations (e.g. +, - , x, ÷). Matrix Algebra is a collection of numbers ordered by rows and columns. It is regular to enclose the elements of a matrix in parentheses, braces, or brackets. Matrix is symbolized by bold face capital letters enclosed by brackets or parentheses. A matrix that has two rows and three columns is referred to as a 2x3 matrix. The genesis of the matrices dates back to end of the 17th Century when the idea reappeared, and development really got underway (Abadir and Magnus 2005). The origin of matrices and determinants rose through the study of systems of linear equations.

Matrix Algebra applies to several branches of science and different mathematics disciplines (Harville 1997). Matrices present a most suitable vehicle for organizing and storing large quantities of data. A matrix is a tool for organizing vast quantities of data. Matrices are used to represent complex systems and operations by compact entities. Matrix representations are possible in various ways, hence, there is transportation matrix, distance matrix, and cost matrix.

Matrix Algebra also assists us in calculating the electrical properties of circuits with voltage, amperage or resistance. The field of probability and statistics may also use matrix representation. A probability vector will list the probabilities of different outcomes of one trial. Computers run Markov simulations based on stochastic matrices in order to model events ranging from gambling through weather forecasting to quantum mechanics. A stochastic matrix is a square matrix whose rows are probability vectors (Abadir and Magnus 2005).

Matrix Algebra can be used in a wide variety of decision situations. Perhaps its widest use is in examining and predicting the behavior of consumers in terms of their brand loyalty and switch over from one brand to another (Searle 1982). Another application of this technique in business was used towards the study of accounts receivable behavior: that is, the study of consumers as they change from "current account" through "30 days overdue" to "30 to 60 days overdue" and then to "bad debt." In each of these applications, the interest is in predicting what the future will bring (number of bad debts, for example, in the account receivable application) by analyzing what the present behavior is—the propensity of customers to move from one current account to various past due categories (Searle 1982).

Maximum Likelihood Estimation

Maximum Likelihood Estimation is described as a simple method of constructing an estimator for an unknown parameter. The method is important because it gives a single value for the unknown parameter (Orloff and Bloom 2014:). Hurlin sees the method as a tool for estimating the parameters of a model. This estimation method is one of the most widely used to select the set of values of a model's parameters that maximizes the likelihood function. Intuitively, this maximizes the "agreement" of the selected model with the observed data. The Maximum Likelihood Estimation Method gives a unified approach to estimation.Italways begins with a mathematical expression known as a "likelihood function" of the sample data. This expression contains the unknown parameters to be estimated. Those values of the parameter that maximize the sample likelihood are known because the maximum likelihood estimates which are determined by setting the partial derivative of the likelihood function to zero: that is, finding the location of the function's peak with respect to the estimated parameters (Hurlin 2013:).

The Maximum Likelihood Estimation Method is applied through means, variances, probabilistic distribution, and covariance of the observations. The method is a robust parameter judgment technique. These parameters depend on the assumptions. In fields like finance, a small number of very large observations are of crucial importance for the computation of some quantity of interest. For instance, the estimation of a large quantile (say 99% or more) depends heavily on few observations in the right tail. In this case, it is extremely important to choose a probabilistic model that accounts for these observations (Bee et al. 2009). The method can also be employed in logistic regression to estimate unknown parameters: for instance, to find the set of parameters for which the probability of the observed data is greatest (Czepiel 2010).

The maximum likelihood estimation of the parameters of a Pareto mixture is an example of this methodology. Bee et al. (2009:) demonstrate that the Pareto distribution is a commonly used model for heavy-tailed data. It is a two-parameter distribution whose shape parameter determines the degree of heaviness of the tail, so that it can be adapted to data with different features. The Pareto distribution was introduced by the Italian economist Pareto as a model for income distribution, and it has subsequently been used mostly as a model for heavy-tailed data, in particular in hydrology, insurance, and finance.

Metric Scaling

A metric scale is a form of measurement used in the metric system. The metric system is the world standard for measurement and is made of three basic units: (1) meter, (2) gram, and (3) liter. The meter measures length, the gram measures mass, and the liter measures volume) (reference.com).

The Metric Scaling Method is employed to summarize interval scales, ratio scales, and absolute scales (Kenkel 1986). The metric scale is the system of measurement used in the metric system. Unlike the customary system of measurements in the United States, the metric scale has units based on multiples of 10.

Different metric scales exist for different types of measurements. The meter is the metric scale unit for length; the gram is the metric scale unit for the mass of weight, which is weight divided by force of gravity; and the liter is the metric scale unit for volume. One advantage of the metric scale is that it makes it easier for users to perform calculations because, as stated above, its units are based on multiples of 10 (sciencing.com).

Multivariate Analysis of Variance

Tabachnick and Fidell (1996) and Pearl (2000) define the Multivariate Analysis of Variance (henceforth, MANOVA) Method simply as an Analysis of Variance (henceforth, ANOVA) with several dependent variables. Put simply, ANOVA tests for the difference in means between two or more groups, while MANOVA tests for the difference in two or more vectors of means.

George and Dunteman (1984) presented a perfect example of the application of MANOVA on students' improvements in math and physics. An example of how MANOVA can be utilized, according to these scholars, is to conduct a study in which two different textbooks are used try to determine students' improvements in math and physics. In this case, improvements in math and physics were the two dependent variables, and the research hypothesis was that both variables together were affected by the difference in textbooks. MANOVA could then be used to test this hypothesis (George and Dunteman 1984).

Instead of a univariate F value/statistic (i.e. "the variance of the group means/ mean of the within group variances"), one would obtain a multivariate F value (Wilks' λ) based on a comparison of the error variance/covariance matrix and the effect variance/covariance matrix. Although we only mention Wilks' λ here, there are other statistics that may be used, including Hotelling's trace and Pillai's criterion. The covariance here is included because the two measures are probably correlated, and must take this correlation into account when performing the significance test (George and Dunteman 1984).

According to Cooley and Lohnes (1971), testing multiple dependent variables is accomplished by creating new dependent variables that maximize group differences. These artificial dependent variables are linear combinations of the measured dependent variables. This is better explained by Morrison (1967) in his example with the following five research questions:

1. What are the main effects of the independent variables?
2. What are the interactions among the independent variables?
3. What is the importance of the dependent variables?
4. What is the strength of association between dependent variables?
5. What are the effects of covariates? How may they be utilized?

According to Morrison (1967) and Pearl (2000), the main objective in using MANOVA is to determine if the response variables (student improvement in the example mentioned above) are altered by the observer's manipulation of the independent variables. Therefore, their preceding research questions may be answered by using MANOVA.

If the overall multivariate test is significant, we conclude that the respective effect (e.g., textbook) is significant. However, our next question would of course be whether only math skills improved, only physics skills improved, or both. In fact, after obtaining a significant multivariate test for a particular effect or interaction, customarily one would examine the univariate F tests for each variable to interpret the respective effect. In other words, one would identify the specific dependent variables that contributed to the significant overall effect (Morrison 1967; Pearl 2000).

MANOVA is useful in experimental situations where at least some of the independent variables are manipulated. It has several advantages over ANOVA. First, by measuring several dependent variables in a single experiment, there is a better chance of discovering which factor is truly important (Morrison 1967; Pearl 2000). Second, it can protect against Type I errors that might occur if multiple ANOVAs were conducted independently. Additionally, it can reveal differences not discovered by ANOVA tests. However, there are several cautions as well. It is a substantially more complicated design than ANOVA, and therefore there can be some ambiguity about which independent variable affects each dependent variable. Thus, the observer must make many potentially subjective assumptions. Moreover, one degree of freedom is lost for each dependent variable that is added. The gain of power obtained from decreased Sum of Square Error (SSE) may be offset by the loss in these degrees of freedom (Morrison 1967; Pearl 2000). Finally, the dependent variables should be largely uncorrelated. If the dependent variables are highly correlated (i.e. multicollinearity), there is little advantage in including more than one in the test given the resultant loss in degrees of freedom. Under these circumstances, use of a single ANOVA test would be preferable (Morrison 1967; Pearl 2000).

Neural Networks

Neural Networks (henceforth, NNs) also known as Artificial Neural Networks (henceforth, ANNs) are an information processing paradigm that is inspired by how biological nervous systems, such as the brain, process information (Stergious and Siganos n.d.). It is made up of many highly interconnected processing elements (neurones) that work together to solve specific problems by trying to imitate the structure and the function of the human nervous system. An NN is configured for a specific application, such as pattern recognition or data classification, through a learning process. In simple terms, it is a computer system modelled on the human brain and nervous system. This system however differs from the conventional computers in terms of the problem-solving approach. Conventional computers use an algorithmic approach where the computer follows a set of instructions in order to solve a problem. The NN resembles the human brain in two ways: (1) it requires knowledge through learning; and (2) knowledge is stored within the interconnection strengths known as synaptic (Stergious and Siganos n.d.).

The neuron is a device with many inputs and one output. A neuron has two modes of operation: (1) the training mode and (2) the using mode. In the training mode, the neuron can be trained to fire (or not) for particular input patterns. In the using mode, when a taught input pattern is detected at the input, its associated output becomes the current output. If the input pattern does not belong in the taught list of input patterns, the firing rule is used to determine whether to fire or not (Stergious and Siganos n.d.).

The firing rule is an important concept in neural networks and accounts for their high flexibility. A firing rule determines how one calculates whether a neuron should fire for any input pattern. It relates to all the input patterns, not only the ones on which the node was trained. After the firing rule, the patterns are recognized and implemented by using a feed-forward neural network that has been trained accordingly. During training, the network is trained to associate outputs with input patterns. When the network is used, it identifies the input pattern and tries to output the associated output pattern (Stergious and Siganos n.d.).

NNs have been successfully applied in several industries such as for sales forecasting, industrial process control, customer research, data validation, risk management, and target marketing. In medicine, it has been used to recognize diseases such as tumors, bony structures, fractures and infections in the organs and tissues using various scans such as computerized tomography scan and ultrasonic scans (Stergious and Siganos n.d.).

Methodology

This is a method used in psychology and the social sciences to study people's subjectivity or viewpoints. It was developed by William Stephenson in 1993/1994. This method is widely used in clinical settings to assess a patient's progress over time or examine how people think about a topic: i.e. inter-rater comparisons (Stephenson 1993/1994).

Q-Methodology comes from the form of factor analysis that is used to analyze data. While the R-Methodology involves finding correlations between variables—e.g., height and age, Q-Methodology looks for correlations between subjects across a sample of variables. Q-Methodology reduces the many viewpoints of the subjects to a few factors which represent shared ways of thinking. Statements of a Q sort are drawn from a "concourse": i.e. the sum total of all things people say or think about a subject. The Q sort scaling is a rank technique whereby respondents are asked to sort presented objects into piles based on similarity according to a specified criterion such as preference, attitude, or perception. It typically uses fewer subjects (sometimes only one) as compared to other methods. Another advantage of the methodology is that it is not expensive (Stephenson 1993/1994).

In studies of intelligence, the Q factor generates consensus-based assessment scores. The process usually involves using a paper template and a sample of statements printed on individual cards (Stephenson 1993/1994). An application of the Q-Methodology would be in identifying and categorizing the opinions of primary care physicians and students on acceptance or resistance to adapting information technologies in the health care workplace (Stephenson 1993/1994).

Randomized Responses

The Randomized Responses Method is a survey technique proposed by Warner (1965)to "reduce potential bias to non-response and social desirability when asking questions about sensitive behaviors and beliefs" (Warner 1965: 63). In other words, this method is used to protect and ensure confidentiality of respondents, particularly when the information needed, or question asked is sensitive (Rueda, Arcos and Cobo 2015). Thus, this method becomes an important survey tool to gain the cooperation and confidence of respondents in orderto obtain valid and reliable information to avoid biased responses (Blair, Imai and Zhou 2015; Rueda, Arcos and Cobo 2015). It means that this method precludes the challenges of untruthful responses when sensitive and personal questions are asked of respondents. The method also prevents respondents from "over-reporting of socially desirable behavior and attitude and systematic under-reporting of socially undesirable ones" (Blair, Imai and Zhou 2015: 1304).

By extension, the method dictates the respondent's answered question. To achieve this, the method allows the respondent the opportunity to select the question s/he will answer using randomized maneuvers such as the toss of a coin (Blair, Imai and Zhou 2015; Rueda, Arcos and Cobo 2015).

Regression Analysis

Regression Analysis is a statistical method for estimating the relationships among variables. It is a method for measuring the link between two or more phenomena being examined in a study. It helps one to understand the typical value of the dependent variable, how it changes when any one of the independent variables is varied while other independent variables are fixed. Regression Analysis is used to predict, forecast, and understand which ofthe independent variables are related to the dependent variable(s) and to explore their relationships. Additionally, it is useful for analyzing large amounts of data (Gallo 2015).

There are many techniques for carrying out Regression Analysis like simple linear regression, non-linear regression, and multiple regression. The regression equation has (a) y or dependent variable, which represents the process you are trying to predict or understand and it appears on the left side of the equal sign; (b) λor alpha, which is the nonrandom structural component; (c) independent/explanatory variables x, which are variables used to model or predict the dependent variable values, and they appear on the right side of the equal sign; (d) the regression coefficient β, which is computed by the regression tool—coefficients are values, one for each explanatory variable that represents the strength and type of relationship the explanatory variable has with the dependent variable; and (e) ε or error term, which is the disturbance, remainder or residual term. The regression equation is represented as follows:

$$Y= \alpha\ \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \ldots \beta_n X_n + \varepsilon$$

To conduct a Regression Analysis, a researcher is to gather data on the variables, put the information on a chart where the y-axis takes the dependent variable and the x-axis takes the independent variable(s), and draw a line that runs through the middle of the data points to show the relationship between the dependent and independent variables. If working with a statistical program like SPSS, it will provide the error term that refers to the fact that regression is not perfectly precise (Gallo 2015).

Survey Method

A survey is any activity that gathers information in a systematic manner about a situation, an area of interest, or about people's attitudes, opinions, behaviours, interests, or practices. The Survey Method is predominantly descriptive. This explains why the terms descriptive and survey method are used interchangeably (Fowler 2013). The method involves the collection of data from a sample of elements (e.g., adult women) drawn from a well-defined population (e.g., all adult women in the Maasai community) using a questionnaire. According to Pinsonneault (1993), surveys are good for gathering information about the characteristics, actions, or opinions of a large group of people.

Written surveys are best suited for confidential information. This approach is often used in military research where action reports are used to evaluate an exercise (Chilisa, Bagele and Preece 2005). Verbal surveys through face-to-face interviews are useful where the actual population is not known or where there is respondent resistant to written surveys. A mixed method of both written and verbal surveys reflects a higher composite response rates than single medium survey (Salant, Priscilla and Don 1994).

An example of the effective utilization of the Survey Method is the case of Botswana. Surveys in which everybody is required to participate are the population census, livestock census, and agricultural census. The law states that a person who without reason refuses to participate in a survey shall be guilty of an offence and liable to a fine or imprisonment or both (Chilisa 2005).

Test Item Bias

Bias refers to the characteristics of an item that might lead to differential performance for individuals of the same ability. Ideally, in a good test, learners with similar knowledge of the test should perform similarly on individual examination items, regardless of gender, culture, ethnicity, or race (Perrone 2006).When used in education, a biased test is one whose test design, or the way results are interpreted and used, systematically marginalizes a group of learners over others (Hambleton & Rodgers 1995).

There are three types of methods used to test for item bias: (1) construct validity, (2) content validity, and (3) predictive validity. The construct-validity bias test method investigates the ability of a given test to accurately enumerate what it was developed to measure. An example would be a mathematics intelligencetest that has mathematical concepts that students have not learned. Consequently, their test results might reflection their weak mathematical competencies rather than academic or intellectual abilities (Perrone 2006; Hambleton & Rodgers 1995).

The content-validity bias test is employed when a single test in terms of content becomes comparatively more difficult for one group of students than for others. This could be a result of existing bias in the exposure of different subcategories to the learning materials available. Another example is when the scoring disadvantages a group of students over others. Some of the questions might have varying answers or even conflicting answers based on their varying cultural context; yet, the scoring sheet deems one of the group's answers incorrect. In some tests, some questions are worded in ways that are unfamiliar to certain students because of linguistic or cultural differences (Perrone 2006; Hambleton & Rodgers 1995).

The predictive-validity bias test, sometimes referred to as criterion-related validity, is used to look at an ability of a test to accurately predict the performance of a student in future. Unbiased tests are those tests that can predict the academic performance of students in future performance (Perrone 2006; Hambleton & Rodgers 1995).

In sum, test scores bias is a theoretical concept. It depends on a theoretical notion of a true score. Nonetheless, the existence of differences in group test scores does not necessarily mean that test scores are biased. Sometimes, the group differences may reflect the real difference in the test.

Test of Significance

The Test of Significance is a statistical method employed to draw inferences from samples about a population. Therefore, the significance level for a given hypothesis test is a value for which a P-value less than or equal to is considered statistically significant. A significance test is performed to determine if an observed value of a statistic differs enough from a hypothesized value of a parameter to draw the inference that the hypothesized value of the parameter is not the true value (Lane and Dunlap 1978).

When we use a test of significance to compare two groups, we usually start with the null hypothesis that there is no statistical difference between the populations from which the data come. If this hypothesis is not valid, the alternative hypothesis must be true that there is a difference. When the null hypothesis is rejected, the effect is said to be statistically significant. If the null hypothesis is rejected, then the alternative to the null hypothesis (called the alternative hypothesis) is accepted (Bland and Bland 1994). Steps involved in testing for statistical significance include stating the null hypothesis, selecting a probability of error level (alpha level), selecting and computing the test for statistical significance, and interpreting the results (Saint-Germain 2010).

The Test of Significance Method is used to estimate the probability that a relationship observed in the data occurred only by chance; this means that the probability that the variables are unrelated in the population is tenable. The method has also been employed to filter out unpromising hypotheses. Tests for statistical significance are used because they constitute a common yardstick that can be understood by a great many people, and they communicate essential information about a research project that can be compared to the findings of other projects (Saint-Germain 2010).

Time Series Analysis

Time Series Analysis involves the application of methods to naturally ordered series data to reveal their characteristics and statistics (Strickland 2015). It may include regularly or sequentially recorded data. Two methods are mainly used: (1) frequency-domain methods and (2) time-domain methods. Frequency domain methods are divided into spectral analysis and wavelet analysis. Wavelet analysis uses auto-correlation and cross-correlation analyses. Time domain methods use scaled correlation. Other techniques include parametric and non-parametric methods. Parametric methods set out the parameters of structures in the stochastic process. Non-parametric methods do not assume a structure within the data by estimating covariance or spectrum. Furthermore, time analysis categories may include univariate and multivariate and linear and non-linear (Strickland 2015).

Also, using previously observed data, future values can be predicted (i.e. time series forecasting). It can be applied "to real-valued, continuous data, numeric data, or discrete symbolic data: i.e. sequences of characters, such as letters and words in the English language" (Strickland 2015). An example of time series data includes the recording of daily closings of a stock exchange. The Time Series Analysis Method was used to analyze stock prices by Green (2011) using the Box-Jenkins approach: i.e. taking value samples at regular intervals and enabling the identification of autocorrelation patterns within the data.

UnidimensionalScaling

The Unidimensional Scaling Method is the special one-dimensional case of multidimensional scaling which provides a visual representation of the pattern of proximities: for example, similarities or distances among a set of objects. It is often discussed separately because the unidimensional case is quite different from the general multidimensional case. It is applied in situations where we have a strong reason to believe that there is only one interesting underlying dimension, such as time, ability, or preference. The method is also very different from multidimensionalscaling techniques because the latter use very different algorithms to minimize their loss (De Leeuw 2005).

Thus, the unidimensional scaling problem, with a known scale order, requires us to minimize $(x - t)0V(x - t)$ over all x satisfying the order restrictions. This is a monotone regression problem [2], which can be solved quickly and uniquely by simple quadratic programming methods (De Leeuw 2005).

Now for some geometry, the vectors x satisfying the same set of order constraints form a polyhedral convex cone K in Rn. Think of K as an ice cream cone with its apex at the origin, except for the fact that the ice cream cone is not round, but instead bounded by a finite number of hyper planes. Since there are $n!$ different possible orderings of x, there are $n!$ cones, all with their apex at the origin. The interior of the cone consists of the vectors without ties and intersections of different cones are vectors with at least one tie. Obviously, the union of the $n!$ cones is all of Rn. Thus, the unidimensional scaling problem can be solved by solving $n!$ monotone regression problems, one for each of the $n!$ cones. The problem has a solution which is at the same time very simple and prohibitively complicated. The simplicity comes from the $n!$ sub-problems, which are easy to solve, and the complications come from the fact that there are simply too many different sub-problems. Enumeration of all possible orders is impractical for *n* > 10, although using combinatorial programming techniques makes it possible to find solutions for *n* (De Leeuw 2005).

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